References

[1]Pines, S., "An Elementary Explanation of the Flutter Mechanism," *Proceedings of the National Specialists Meeting on Dynamics and Aeroelasticity*, Fort Worth, Texas, Nov. 1958, pp. 52-58.

[2]Landahl, M. T., "Graphical Technique for Analyzing Marginally Stable Dynamic Systems," *Journal of Aircraft*, Vol. 1, May 1964, pp. 293-299.

[3]Ferman, M. A., "Conceptual Flutter Analysis Techniques," McDonnell Company, Rept. F322, Feb. 1967.

[4]Pines, S. and Newman, M., "Structural Optimization for Aeroelastic Requirements," AIAA Paper 73-389, Williamsburg, Va., March 1973.

[5]Stark, V.J.E., "Application to the Viggen Aircraft Configuration of the Polar Coordinate Method for Unsteady Subsonic Flow," *The 9th Congress of the International Council of the Aeronautical Sciences*, ICAS Paper 74-03, Haifa, Aug. 1974.

Fracture Mechanics Correlation for Tensile Failureof Filamentary Composites with Holes

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Introduction

HOLES in filamentary composite materials are of great and continuing interest, since mechanical fasteners are important for joining components together. Holes in ductile metals do not affect the static tensile strength, because the plasticity of the stress-strain behavior smooths out the stress distribution at failure. Since such filaments as boron and graphite are inherently brittle, almost all new designs must first determine the failure characteristics of the laminate layup with holes.

Waddoups et al.[1] presented results indicating that filamentary composite materials with holes exhibited a behavior similar to that exhibited by ductile metals with cracks. However, in order to use linear elastic fracture mechanics, they postulated the existence of intense energy regions at the edges of the hole in which the fracture process takes place. These intense energy regions are modeled as edge cracks emanating from a circular hole, and then linear elastic fracture mechanics is applied. With this model, Waddoups et al.,[1] by using the Bowie solution for stress intensity, defined a fracture toughness K_Q and demonstrated a correlation with the hole size.

Whitney et al.[2] have formulated a stress fracture criterion that assumed that failure occurs when the stress at a distance d_0 ahead of the hole reaches the tensile strength σ_0 of the virgin material (no hole). They also have an alternate criterion that assumes that failure occurs when the average stress over a distance a_0 ahead of the hole reaches the virgin strength. The effect of hole size is encompassed in these criteria, because the stress distribution in the vicinity of the hole explicitly contains the size of the hole.

All of the above theories correlate reasonably well with experimental data and appear to be adequate for purposes of engineering design, given sufficient experimental data. The above theories implicitly assume the material to be homogeneous and then apply linear elasticity to the devleopment of what might be called a "macroscopic" theory of failure.

This Note proposes a "microscopic" theory, which postulates that fracture of a filamentary composite initiates at a crack lying in the matrix material at the interface of the matrix/filament (thus explicitly recognizing the filament and matrix as two different materials). The effect of hole size is then shown to correlate with the singularity at the tip of this crack, which is at the interface of two different materials.

Theory

The motivation for this theory comes from linear elastic fracture mechanics as applied to homogeneous materials where the fracture stress σ_f in the presence of a crack is given by

$$\sigma_f \propto K_{IC}(2a)^{½} \tag{1}$$

Thus, K_{IC} is a material parameter called the fracture toughness, which is experimentally determined and which has dimensions of stress × length to the one-half power. The crack size effect ($2a$ is the length of the crack) is embodied in the exponent of $-½$, which is the order of the mathematical singularity at the tip of the crack.

The present theory proposes an equation for the fracture stress σ_f of filamentary composites of the form

$$\sigma_f = H(2a)^{-m} \tag{2}$$

where H is akin to K_{IC} but has dimensions of "stress × length to the m power," which is different than that for K_{IC}. The exponent $-m$ is the order of the singularity of a crack with its tip at the interface of two different materials. It has been shown that the order of the singularity is a function of the ratio μ_1/μ_2 of the shear moduli of the two materials and of the two Poisson ratios ν_1 and ν_2.[3,4] Table 1 gives values of this singularity for a selected range of parameters.

Not much statistical data on the shear moduli of boron filaments, graphite filaments, and the epoxy resins are available. If the shear moduli of the boron and 6061 aluminum are taken as 24×10^6 and 3.8×10^6 psi, respectively, then the ratio of moduli for the boron/aluminum system is approximately 0.16. If the graphite filament and the epoxy resin tensile moduli are taken as 28×10^6 and 0.7×10^6 psi, respectively, then the ratio is approximately 0.025.

Experimental Justification

In Fig. 1, fracture data from several different experiments are shown plotted in the format of $\log \sigma_f/\sigma_0$ versus $\log 2a$. It is be to observed that two different material systems are shown. One system, consisting of experimental data[8] obtained during the course of this investigation, is the 5.6-mil-diam boron

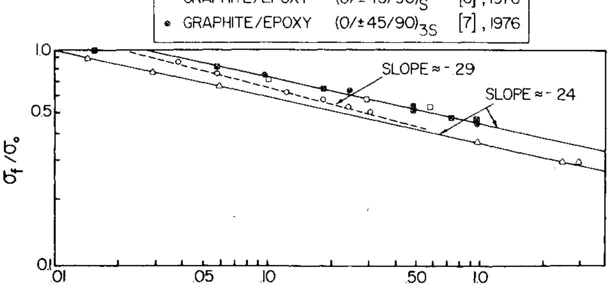

Fig. 1 Experimental data.

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Table 1 Order of singularity

μ_1/μ_2	ν_1	ν_2	m
.01	.30	.20	.250
.01	.35	.20	.269
.025	.30	.15	.263
.025	.30	.20	.262
.025	.30	.25	.261
.025	.35	.20	.280
.050	.30	.20	.280
.050	.35	.20	.297
.10	.30	.20	.310
.158	.30	.20	.339
.20	.30	.20	.357
.10	.33	.20	.319
.158	.33	.20	.347
.20	.33	.20	.364

filament in a 6061 aluminum matrix. The other system[5-7] is graphite filament in an epoxy matrix. For the graphite/epoxy system, data from two different ply layups are presented. All of the experimental data have been adjusted by means of a correction factor to appear as if the specimens were infinitely wide.[8]

Three straight lines have been fitted (by eye) to the experimental data. One line goes through the boron/aluminum data, one goes through the three different sets of graphite/epoxy (0/±45/90) data, and the third goes through the graphite/epoxy (0/±45) data. The slope of the boron/aluminum data is approximately −0.29, whereas the slopes of the other two lines are −0.24. According to the proposed theory the slope of these lines in a log-log plot should be the order of the singularity.

As can be seen by examining Table 1, the most likely value of the singularity for the graphite/epoxy system is approximately 0.26. For the boron/aluminum system the most likely value is about 0.33. The term "most likely" is used because accurate data on shear moduli and especially on Poisson's ratios are not available.

Conclusions and Observations

1) As predicted by the theory, the slope of the boron/aluminum line is steeper than for the graphite/epoxy line.

2) The experimental slopes are not as steep as predicted by the theory.

3) It may be possible to correlate the factor H in Eq. (3) to the number of zero-degree plies of the layup. At any hole size, $2a$, the ratio of the respective σ_f/σ_0 values for the two graphite/epoxy layups is about 0.80 (i.e., the ratio of their respective H values). The ratio of 0° plies in the two graphite/epoxy layups is 0.75, i.e., one-third of the plies in the (0/±45) layup is oriented at 0°, whereas one-fourth is the proportion for the (0/±45/90) layup. Obviously, much more experimental work is required before such a simple correlation can be used.

4) The filaments in the ±45° plies in a layup contribute to the "toughness," whereas the ones at 90° do not because fracture can occur solely in the matrix of the 90° plies. This may be a partial explanation for the value of 0.80 against 0.75 in the previous paragraph.

5) Implicit in the proposed correlation is the assumption that cracks in the matrix perpendicular to the 0° filaments are the triggering mechanism for the final failure. Cracks in the matrix at an angle to the 45° filaments have a different kind of a singularity[4] and hence induce an intensity of stress in these filaments insufficient to initiate catastrophic failure in the layup.

6) It would be interesting to conduct experiments in laminates with the nonzero plies at smaller than 45° to determine the slope of the line and to correlate this slope against the theoretical singularity.

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References

[1] Waddoups, M. E., Eisenmann, J. R., and Kaminski, B. E., "Macroscopic Fracture Mechanics of Advanced Composite Materials," *Journal of Composite Materials*, Vol. 5, Oct. 1971, pp. 446-454.

[2] Whitney, J. M. and Nuismer, R. J., "Stress Fracture Criteria for Laminated Composites Containing Stress Concentrations," *Journal of Composite Materials*, Vol. 8, July 1974, pp. 253-265.

[3] Zak, A. R. and Williams, M. L., "Crack Point Singularities at a Bi-Material Interface," *Journal of Applied Mechanics*, Vol. 85, Series E., March 1963, pp. 142-143.

[4] Lin, K. Y. and Mar, J. W., "Finite Element Analysis of Stress Intensity Factors for Cracks at a Bi-Material Interface," *International Journal of Fracture*, Vol. 12, Aug. 1976, pp. 521-531.

[5] Nuismer, R. J. and Whitney, J. M., "Uniaxial Failure of Composite Laminates Containing Stress Concentrations," *Fracture Mechanics of Composites*, ASTM STP 593, 1975, pp. 117-142.

[6] Daniel, I. M., "Biaxial Testing of Graphite/Epoxy Composites Containing Stress Concentrations," *Mechanics of Composites Review*, USAF AFML, USAF AFOSR, Dayton, Ohio, Oct. 1976, pp. 123-151.

[7] Walter, R. W., Johnson, R. W., June, R. R., and McCarthy, J. E., "Designing for Integrity in Long Life Composite Aircraft Structures," ASTM Symposium on Fatigue of Filamentary Composite Materials, Nov. 1976.

[8] Lin, K. Y., "Fracture of Filamentary Composite Materials," Ph.D. Dissertation, Department of Aeronautics and Astronautics, MIT, Cambridge, Mass., Jan. 1976.